Title: Can You Make a Hole in One?

Brief Overview:

In this activity students will use miniature golf to explore principals of reflection. Students will collect and analyze data to determine the characteristics of the point at which a ball must hit a wall in order to make a hole in one. Students in higher level math classes (geometry, trigonometry, or calculus) will use equations to compare the actual data with what should theoretically happen.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  The students will use a variety of approaches to solve the basic problem and then choose a method that is best for other problems.

- **Mathematics as Communication**
  The students will be able to explain the methods used in data collection. They will be able to explain how to find the point where the ball must hit the bumper to make a hole in one for new holes.

- **Mathematics as Reasoning**
  The students will make predictions about how changing the hole position and the tee position will affect the point at which they must hit the bumper.

- **Algebra**
  The students will use the TI-83 to find equations that fit their data. They will use graphs to find minimums and points of intersection.

- **Geometry from a Synthetic Perspective**
  The students will construct scale drawings and find the point to make a hole in one by constructing a virtual hole on the other side of the bumper.

- **Geometry from an Algebraic Perspective**
  The students will use the basic trigonometric functions and equations using the Pythagorean theorem to create models of the length of the path of the golf ball and the angle at which it hits the bumper.

- **Trigonometry**
  The students will use trigonometric functions to analyze the path and angle of the golf ball necessary to make a hole in one.

- **Conceptual Underpinnings of Calculus**
  The students will be introduced to discussions of minimum and maximum values in extensions of the problems.

Links to Science Standards:

- **Science as Inquiry**
  The students will be conducting a scientific experiment involving reflection. They will collect data and compare it to theoretical predictions. They will discuss factors which caused actual values to vary from theoretical predictions.

Links to Maryland High School Mathematics Core Learning Goals

- **1.1**
  The students will analyze a wide variety of patterns and functional relationships using the language of mathematics and appropriate technology. **(1.1.1, 1.1.2, 1.1.4)**

- **1.2.4**
  The students will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

- **2.1.4**
  The students will validate properties of geometric figures using tools and technology.

Grade/Level:

This problem is appropriate for grades 9-12 and classes from applied algebra through trigonometry.

Duration/Length:

Depending on the level of the math class and on the extent to which the activities are pursued, this activity could take as little as three traditional classes or as many as five block classes.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Basic measurement with a ruler and protractor

Objectives:

Students will:

- use a TI graphing calculator to record and analyze data using stat list, stat plot, and curve fitting techniques.
- be able to discuss the characteristics of the point at which the ball must hit the bumper (i.e. the angle of incidence is equal to the angle of reflection at this point and the shortest path to the hole passes through this point).
- make scale drawings of the golf hole and find the point where the ball must hit the bumper by synthetic geometry (geometry).
- be able to explain why actual data did not match theoretically predicted results.
- write equations to determine the path length of the ball and the angle of incidence for any distance along the bumper (algebra, geometry, trigonometry).
- explain two or three ways to find the point at which the ball must hit the bumper to make a hole in one for a different hole.

Materials/Resources/Printed Materials:

- Golf ball and putter
- 2”X4” 10’ in length
- Cup for hole
- Tape to mark tee and inelastic string at least 16’
- Tape measure, ruler, and protractor

- Graphing calculators
- Laser pointer and small mirror (optional)

Development/Procedures:

Before actually playing miniature golf, discuss what students already know about a ball bouncing off a wall/bumper. Develop the necessary vocabulary including angle of incidence and angle of reflection. Assess whether the class already knows that the angle of incidence should equal the angle of reflection. Discuss what may prevent a ball from bouncing as it should perhaps by relating to basketball or pool shots. Compare these situations to the golf ball and board. Also, discuss other real-world examples of reflection including a light and mirror.

Since students will be using the TI calculator to record and analyze data; the teacher should be prepared to review the TI or to walk the class through data recording and analysis. If the class is working in small groups, be prepared to work with composite data.

Higher level math classes will be working with scale drawings or sketches of the problem to find the point where the ball must hit the bumper to make a hole in one by construction or by equations. This could be done before or after the golf and data collection.

The materials suggested for this activity are minimal in terms of lumber. Enterprising teachers may want to build a more elaborate and realistic looking golf hole and have materials to construct other configurations

Performance Assessment:

The teacher should supervise the groups as they collect their data. The activity sheets can be assessed for the accuracy of the measurements. The analysis of data can be assessed by how close the students come to the theoretical result. Scale drawings can be judged for accuracy and equations can be checked for accuracy. Most of the extension activities are performance assessments.

Extension/Follow Up:

- Students may draw their own golf holes and have other students determine how to play their hole successfully.

- Students may construct other golf holes in the classroom and determine how they should be played by a variety of methods.

- Students may explore the relationship between the relative distance of the hole and tee from the bumper to the location of the point of impact along the bumper.

- Higher level classes may explore the situation in which two reflections are necessary for a hole in one.

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CAN YOU MAKE A "HOLE IN ONE" ? ? ?

Lesson 1

DESCRIPTION:

An analysis of mathematical concepts involved in the game of miniature golf. Lesson 1 features reflection from a bumper which is positioned parallel to the direct path from the tee to the hole.

MATERIALS:

REQUIRED

1 Self adhesive sticker to mark the tee.
1 Styrofoam coffee cup to simulate the cup.
1 Empty soft drink can as an obstruction in front of the cup.
1 Piece 2X4 lumber twelve feet long to serve as a bumper
1 Piece of string (inelastic) at least sixteen feet in length.
1 measuring tape at least fifteen feet in length.
1 protractor
1 golf ball
1 putter
Graphing calculator (TI-83)

OPTIONAL

Laser light pointer
Mirror

DIRECTIONS AND SETUP:

1. Set 2X4 bumper on the floor firmly against a wall.

2. Mark 2X4 bumper in one foot increments. Label the one foot distance mark as zero, the two foot distance mark as one, the three foot distance mark as two, and so forth until the last distance marking is labeled ten.

3. Place the sticker representing the tee on the floor exactly perpendicular from the bumper at distance mark zero, exactly four feet from the bumper.

4. Place the cup exactly perpendicular from the bumper at distance mark 10, centered exactly four feet from the bumper.

5. Place the obstruction exactly perpendicular from the bumper at distance mark 8, centered exactly four feet from the bumper. This obstruction is placed so that a direct shot from the tee to the cup is not allowed. The placement of the obstruction requires the shot to be banked off the bumper.

6. Have students enter distance label numbers zero through ten in list one in their graphing calculators.

HYPOTHESIS:

1. Have students guess the distance mark on the bumper at which they must aim to make the "HOLE IN ONE"

2. Have students write a justification for their guess.

EXPERIMENTATION:

1. Have students volunteer to test their hypothesis by banking the golf ball off the designated mark on the bumper. Ensure at least one test for each different guess in the class.

2. Have students record the results of each trial test. Rank the results for accuracy.

DATA COLLECTION:

1. Have students stretch the string from the tee to the bumper at distance mark zero, and then on to the center of the cup. Measure the length of the string and record this path length measurement as the zero entry in list two in the graphing calculator.

2. While the string is in place, with a protractor, measure the angle between the bumper and the string going to the tee (90°). This is the angle of incidence. Record this measurement as the zero entry in list three in the graphing calculator.

3. While the string is in place, with a protractor, measure the angle between the bumper and the string going to the hole (21.8°). This is the angle of reflection. Record this measurement as the zero entry in list four in the graphing calculator.

4. Repeat the path length and two angle measurements noted in steps one through three above for each distance mark on the bumper entering the data into lists two, three, and four respectively, in the graphing calculator.

ANALYSIS:

1. Graph a stat plot of the distance (L1) vs the path length (L2)

2. Find the curve of best fit which approximates the data.

3. Use the stat calc function to find the minimum value of the curve.

4. Graph a stat plot of the distance (L1) vs the angle of incidence (L3) AND a stat plot of the distance (L1) vs the angle of reflection (L4).

5. Find the curve of best fit which approximates the data.

6. Use the stat calc function to find the intersection of the two graphs.

7. If several groups have repeated the data collection separately, compile a class average analysis.

DISCUSSION:

1. It has been demonstrated that the shortest path length corresponds to the point where the angle of incidence equals the angle of reflection.

2. Algebra II extension - the distance vs path length plot approximates a parabola with the correct target point corresponding to the minimum value. Calculate path length using Pythagorean theorem.

3. Geometry extension - see supplemental worksheet. Calculate path length using the Pythagorean theorem.

4. Trigonometry extension - calculate angles of incidence and angles of reflection using the tangent function. Calculate path length using the Pythagorean theorem.

5. Proof - Place the laser light pointer on the tee, aimed at the bounce point. Place the mirror flat against the bumper at the bounce point . Adjust the aim of the laser pointer until the reflected beam is centered on the cup. This is the exact bounce point that will result in a “HOLE IN ONE”

6. Identify variables that would cause accurate trials to fail, and inaccurate trials to succeed.

STUDENT DATA COLLECTION SHEET - LESSON ___

NAME ____________________________________

DISTANCE LABEL	PATH LENGTH	ANGLE OF INCIDENCE	ANGLE OF REFLECTION

TEACHER NOTES:

Expected values included below.

Path length entries are calculated using Pythagorean theorem rounded to the hundredth of a foot.When conversion to feet and inches was made the value was rounded to the eighth of an inch.

Angle entries are calculated using the tangent function rounded to the hundredth of a degree - clearly beyond the capability of a protractor.

LIST ONE	LIST TWO	LIST THREE	LIST FOUR
0	14.77 = 14' 9 1/4"	90	21.8
1	13.97 = 13' 11 5/8"	75.96	23.96
2	13.42 = 13' 9 1/4"	63.43	26.56
3	13.06 = 13' 3/4"	53.13	29.74
4	12.86 = 12' 10 3/8"	45	33.69
5	12.81 = 12' 9 3/4"	38.66	38.66
6	12.86 = 12' 10 3/8"	33.69	45
7	13.06 = 13' 3/4"	29.74	53.13
8	13.42 = 13' 9 1/4"	26.56	63.43
9	13.97 = 13' 11 5/8"	23.96	75.96
10	14.77 = 14' 9 1/4"	21.8	90

In lesson two we will investigate the relationships when the bumper is not parallel to the line from the tee to the hole. This is accomplished by moving the hole to some distance from the bumper that is different from the distance between the tee and the bumper.

Geometry Activity

Make a scale drawing of the golf hole on a separate sheet of paper. Place the line representing the bumper near the center of the page. Label the tee as **T** and the hole as **H.** Use a dotted line to represent the path of the ball. Be sure to include your scale with the drawing.

Transformational geometry: Draw the reflection of **H** through the line of the bumper and label it **H'.** Label the point of reflection on the line as **P.**

Euclidian geometry: Draw a line through **H** perpendicular to the line of the bumper labeling the point of intersection **P**. Locate a point **H'** on this line such that **HP=H'P.** This point is called the reflection of **H**.

Draw segment **TH'.** Label the point of intersection with the bumper line **O.**

Answer the following on the paper with your drawing:

What is true of point **O**?

Give a brief explanation of why this is reasonable to you.

Give a short proof (explanation) to show that **ΔOHP** **ΔOH'P.**

What does this congruence suggest about **OH** and **OH'?** About **TH'** and the path of the ball (**TO + OH**)?

Using a new color, draw another ball path from the tee to the hole reflecting off the bumper. Label the point of reflection **O'.** Draw **O'H'**. Why is **TH'** shorter than **TO'+ O'H'**?

Given this result, explain in a sentence why **TO + OH** is the shortest possible path from the tee to the hole reflecting off the bumper.

Extensions:
Use the geometry concepts on this worksheet to explain how a pool player may make an estimate of where to bank a cue ball off a rail in order to hit an object ball.

Given any other miniature golf hole determine if it is possible to make a hole in one by making a shot with one reflection.
Draw a golf hole in which two reflections are required to make a hole in one.

Trigonometry Activity

Make a drawing of the golf hole on a separate sheet of paper. Place the line representing the bumper near the center of the page. Label the tee as **T** and the hole as **H.**
Use a dotted line to represent a path of the ball. Label the point of reflection **O.**

Given:

Write an equation that will give the angle of incidence for any distance **x** along the bumper.

Write an equation that will give the angle of reflection for any distance **x** along the bumper.
(Remember to use **x** to represent the distance from the tee end of the bumper)

Using the graphing calculator, graph both equations and determine the point of intersection.

The angle of incidence is equal to the angle of reflection when **x=________** .

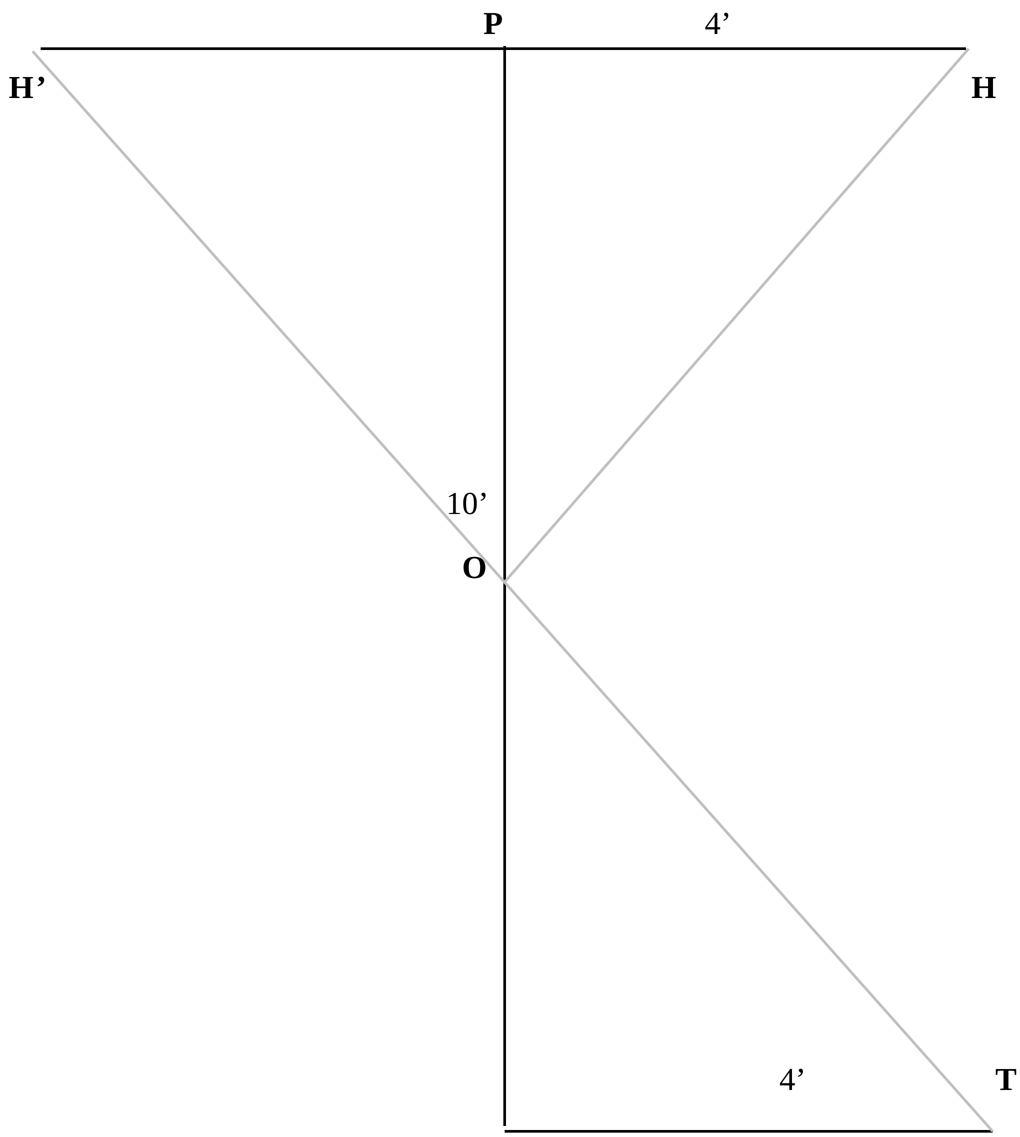
Set the equation for the angle of incidence and angle of reflection equal to each other. Simplify. Based on the simplified equation, explain the relationship between the distances of the tee and hole from the bumper and the location of the point where the angle of incidence equals the angle of reflection along the bumper.

Write an equation for the length of the path that the ball travels using the Pythagorean Theorem.

Graph the equation on the **TI** and find the minimum value of the equation. What is true of the minimum value?

Extension:
Write the equations for other golf holes.

Geometry Activity



ΔOPH   ΔOPH’ since OP=OP, angles at P are rt., and PH=PH’

OH=OH’ cpctc

TH’= TO + OH (the line to the virtual point is the same length as the path of the ball)

Any other path would strike the bumper at O’. TOH’ would form a triangle. In that triangle TH’ would be shorter than the sum of the other two sides (TO’+ O’H’). Since TO’+ O’H’= TO’+ O’H, any other path is longer than the path through the point where the angle of incidence equals the angle of reflection

CAN YOU MAKE A "HOLE IN ONE" ? ? ?

Lesson 2, Part 1

DESCRIPTION:

An analysis of mathematical concepts involved in the game of miniature golf. Lesson 2 features reflection from a bumper which is NOT positioned parallel to the direct path from the tee to the hole.

MATERIALS:

REQUIRED

1 Self adhesive sticker to mark the tee.
1 Styrofoam coffee cup to simulate the cup.
1 Empty soft drink can as an obstruction in front of the cup.
1 Piece 2X4 lumber twelve feet long to serve as a bumper.
1 Piece of string (inelastic) at least twenty five feet in length.
1 measuring tape at least twenty five feet in length.
1 Roll of masking tape to describe a border obstruction.
1 Protractor
1 Golf ball
1 Putter
Graphing calculator (TI-83)

OPTIONAL

Laser light pointer
Mirror

DIRECTIONS AND SETUP:

1. Set 2X4 bumper on the floor firmly against a wall.

2. Mark 2X4 bumper in one foot increments. Label the one foot distance mark as zero, the two foot distance mark as one, the three foot distance mark as two, and so forth until the last distance marking is labeled ten.

3. Place the sticker representing the tee on the floor exactly perpendicular from the bumper at distance mark zero, exactly four feet from the bumper.

4. Place the cup exactly perpendicular from the bumper at distance mark 7, centered exactly ten feet from the bumper.

5. Place a line of masking tape on the floor parallel to and eight feet from the bumper. Begin the line of tape approximately perpendicular to the zero distance mark, end it at a point perpendicular from distance mark five. At distance mark five turn the tape line perpendicular to the bumper and continue until it is at least ten feet from the bumper.

6. Place the obstruction on the corner of the tape line to mark the corner of the area of play.

7. Have students enter distance label numbers zero through ten in list one in their graphing calculators.

HYPOTHESIS:

1. Have students guess the distance mark on the bumper at which they must aim to make the "HOLE IN ONE"

2. Have students write a justification for their guess.

EXPERIMENTATION:

1. Have students volunteer to test their hypothesis by banking the golf ball off the designated mark on the bumper. Ensure at least one test for each different guess in the class.

2. Have students record the results of each trial test. Rank the results for accuracy.

DATA COLLECTION:

1. Have students stretch the string from the tee to the bumper at distance mark zero, and then on to the center of the cup. Measure the length of the string and record this path length measurement as the zero entry in list two in the graphing calculator. Measurements taken in feet and inches (rounded to the nearest one eighth of an inch) should be converted to decimal feet before entry into the graphing calculator.

2. While the string is in place, with a protractor, measure the angle between the bumper and the string going to the tee (90°). This is the angle of incidence. Record this measurement as the zero entry in list three in the graphing calculator.

3. While the string is in place, with a protractor, measure the angle between the bumper and the string going to the hole (55°). This is the angle of reflection. Record this measurement as the zero entry in list four in the graphing calculator.

4. Repeat the path length and two angle measurements noted in steps one through three above for each distance mark on the bumper entering the data into lists two, three, and four respectively, in the graphing calculator.

ANALYSIS:

1. Graph a stat plot of the distance (L1) vs the path length (L2)

2. Find the curve of best fit which approximates the data.

3. Use the stat calc function to find the minimum value of the curve.

4. Graph a stat plot of the distance (L1) vs the angle of incidence (L3) AND a stat plot of the distance (L1) vs the angle of reflection (L4).

5. Find the curve of best fit which approximates the data.

6. Use the stat calc function to find the intersection of the two graphs.

7. If several groups have repeated the data collection separately, compile a class average analysis.

DISCUSSION:

1. It has been demonstrated that the shortest path length corresponds to the point where the angle of incidence equals the angle of reflection.

2. Algebra II extension - the distance vs path length plot approximates a parabola with the correct target point corresponding to the minimum value. Calculate path length using Pythagorean theorem.

3. Geometry extension - see supplemental worksheet. Calculate path length using the Pythagorean theorem.

4. Trigonometry extension - calculate angles of incidence and angles of reflection using the tangent function. Calculate path length using the Pythagorean theorem.

5. Proof - Place the laser light pointer on the tee, aimed at the bounce point. Place the mirror flat against the bumper at the bounce point . Adjust the aim of the laser pointer until the reflected beam is centered on the cup. This is the exact bounce point that will result in a "HOLE IN ONE"

6. Identify variables that would cause accurate trials to fail, and inaccurate trials to succeed.

Lesson 2, Part 2

DESCRIPTION:

Move the cup until it is ten feet from the bumper and perpendicular to distance mark eight.
This will result in the correct bounce point moving to some distance point on the bumper other than the marked integers.

CHALLENGE:

1. Based on the observations in the previous lessons predict the correct bounce point.

2. Explain how you arrived at your prediction.

TEACHER NOTES:

Expected values included below.

Path length entries are calculated using Pythagorean theorem rounded to the hundredth of a foot. When conversion to feet and inches was made the value was rounded to the eighth of an inch.

Angle entries are calculated using the tangent function rounded to the hundredth of a degree - clearly beyond the capability of a protractor.

LIST ONE	LIST TWO	LIST THREE	LIST FOUR
0	16.21 = 16' 2 1/2"	90	55.01
1	15.79 = 15' 9 1/2"	75.96	59.04
2	15.65 = 15' 7 7/8"	63.43	63.43
3	15.77 = 15' 9 3/8"	53.13	68.2
4	16.10 = 16' 1 1/8"	45	73.3
5	16.60 = 16' 7 1/4"	38.66	78.69
6	17.26 = 17' 3 1/8"	33.69	84.29
7	18.06 = 18' 3/4"	29.74	90
8	18.99 = 18' 11 7/8"	26.57	95.71
9	20.25 = 20' 1/2"	23.96	101.31
10	21.21 = 21' 2 1/2"	21.8	106.7

Lesson 2, Part 2

Noting that the correct bounce point is the point where the angle of incidence is equal to the angle of reflection, it is observed that this is where the ratio of the distance from 0 to the bounce point (x) to the distance from the bumper to the tee (4) is equal to the ratio of the distance from the bounce point to the hole (8-x) to the distance from the bumper to the hole (10). Trig students will realize that these ratios are tangent (or cotangent) functions. Even Algebra 1 students can solve the ratio for X to find the bounce point at 2.28

If the students want to measure the distances and angles the following values apply.

LIST ONE	LIST TWO	LIST THREE	LIST FOUR
0	16.81 = 16' 9 5/8"	90	51.34
1	16.32 = 16' 4"	75.96	55
2	16.32 = 16' 1 5/8"	63.43	59
3	16.18 = 16' 2 1/8"	53.13	63.43
4	16.43 = 16' 5 1/8"	45	68.2

The regression curve in the graphing calculator applied to this limited amount of data will produce a curve in which the stat calc function then identifies the minimum value to be 2.36. The more sample points plotted the closer the regression will come to approximating the bounce point of 2.28.